Model-order reduction for the finite-element boundary-element simulation of eddy current problems including rigid body motion

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A coupled finite-element boundary-element method for solving parametric models of eddy current problems is proposed. Affine approximation by the empirical interpolation method makes the numerical model accessible to projection-based parametric model-order reduction. The resulting low-dimensional system provides high evaluation speed at an accuracy comparable to that of the underlying discretization method.

***Index Terms*—Finite element analysis, method of moments, eddy currents, reduced order systems.**

I. Introduction

In the numerical modeling of eddy current problems, such as inductive power transfer systems, one commonly encounters geometrical parameters, especially rigid body motion.

One way to tackle this class of problems is by the finite-element (FE) method in combination with a mesh morphing strategy [1]. This approach imposes some restrictions on the parameter variations, because the morphing method must yield a valid mesh with sufficient element quality. Alternatively, a coupled FE-boundary-element (FE-BE) scheme [2] can be used, which does not require a mesh between the rigid bodies.

The FE-BE method involves assembling and solving a large system of equations. While a single solution may not be expensive on a modern computer, tasks like numerical optimization or response surface modeling over a large parameter domain require large numbers of solutions and are thus time-consuming.

Projection-based methods of parametric model-order reduction (MOR) greatly reduce computational times. However, they require the underlying model to exhibit affine parameterization, which is not the case for the boundary-element (BE) part of the FE-BE method. To make the model accessible to MOR, we propose to approximate Green's function in affine form by means of the empirical interpolation method [3]. The resulting reduced-order model (ROM) features low dimension, and its error is controllable by the size of the ROM.

This paper considers the time-harmonic case only. However, the authors do not foresee any problem with the generalization to the time-domain. In case of non-linearities within the FE region, a suitable MOR method [4] must be applied.

II. Order reduction framework

We consider the $\boldsymbol{A}V$-$\boldsymbol{A}$ FE eddy current formulation [5] in combination with the BE formulation from [2]. The coupled problem for a given parameter configuration leads to an N-dimensional system of linear equations of the form

$$\begin{bmatrix} \mathbf{M}+\mathbf{N} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{FE} \\ \mathbf{x}_{BE} \end{bmatrix} = \begin{bmatrix} \mathbf{b}_{FE} \\ 0 \end{bmatrix}, \tag{1}$$

where $\mathbf{M}$ is a sparse FE matrix, and $\mathbf{B}$, $\mathbf{N}$, and $\mathbf{A}$ are dense BE matrices. The FE excitation vector is denoted by $\mathbf{b}_{FE}$ and the solution vector by $\mathbf{x}_{FE}$ and $\mathbf{x}_{BE}$, respectively. Let $\mathbf{V} \in \mathbb{C}^{N\times n}$, $n \ll N$, be a suitable projection matrix so that

$$\begin{bmatrix} \mathbf{x}_{FE} \\ \mathbf{x}_{BE} \end{bmatrix} \approx \mathbf{V}\hat{\mathbf{x}}. \tag{2}$$

Then, a projection-based ROM for (1) is given by

$$\mathbf{V}^H \begin{bmatrix} \mathbf{M}+\mathbf{N} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{A} \end{bmatrix} \mathbf{V}\hat{\mathbf{x}} = \mathbf{V}^H \begin{bmatrix} \mathbf{b}_{FE} \\ 0 \end{bmatrix}, \tag{3}$$

with the conjugate transpose $(\cdot)^H$. Let $\mathbf{p} \in \mathbb{R}^p$ be a p-dimensional parameter vector that describes rigid body motion. While the FE matrix $\mathbf{M}$ is constant with respect to $\mathbf{p}$, the BE matrices are parameter-dependent:

$$\mathbf{X} = \mathbf{X}(\mathbf{p}), \qquad \text{for } \mathbf{X} \in \{\mathbf{B}, \mathbf{N}, \mathbf{A}\}. \tag{4}$$

So, although the ROM is of low dimension n, its evaluation is independent of the original dimension N only if the left-hand side of (3) is affine in the parameter vector $\mathbf{p}$, i.e., of the form

$$\mathbf{X}(\mathbf{p}) = \sum_i \Theta_i(\mathbf{p})\mathbf{X}_i \tag{5}$$

with scalar-valued functions Θ_i and constant matrices $\mathbf{X}_i$. This is not the case for the BE matrices.

We illustrate the proposed MOR framework by reference to the matrix block $\mathbf{A}$, which is given by

$$[\mathbf{A}]_{ij}(\mathbf{p}) = \iint_{\Gamma\Gamma} \boldsymbol{v}_i(\boldsymbol{x_0}) \cdot \boldsymbol{v}_j(\boldsymbol{y_0}) G(\boldsymbol{x_0}, \boldsymbol{y_0}, \boldsymbol{p})\, \mathrm{d}\Gamma_{\boldsymbol{y}}\, \mathrm{d}\Gamma_{\boldsymbol{x}}; \tag{6}$$

see [2]. Therein, $\boldsymbol{v}_i$ are divergence-free trial functions, and subscript 0 indicates the reference configuration. The parameter-dependence is implicitly contained in Green's function for the Laplace operator

$$G(\boldsymbol{x_0}, \boldsymbol{y_0}, \mathbf{p}) = \frac{1}{4\pi\,|\boldsymbol{f}(\boldsymbol{x_0}, \mathbf{p}) - \boldsymbol{f}(\boldsymbol{y_0}, \mathbf{p})|}, \tag{7}$$

wherein the function $\boldsymbol{f}$ maps the coordinates under rigid body motion. The key idea is to perform an affine approximation of Green's function

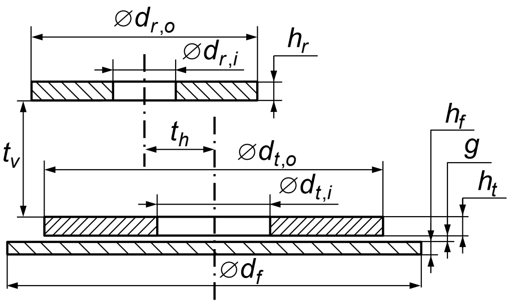


Fig. 1. Sketch of sample WPT system with two litz wire coils. Dimensions in mm: $d_{r,o} = 32.5$, $d_{r,i} = 19.5$, $h_r = 1.5$, $d_{t,o} = 44$, $d_{t,i} = 20.5$, $h_t = 2.1$, $d_f = 50$, $h_f = 2$, $g = 0$, $t_h = 0$. Ferrite: $\mu_r = 850$.

$$G(\boldsymbol{x_0}, \boldsymbol{y_0}, \mathbf{p}) \approx \tilde{G}(\boldsymbol{x_0}, \boldsymbol{y_0}, \mathbf{p}) = \sum_i \Theta_i(\mathbf{p}) q_i(\boldsymbol{x_0}, \boldsymbol{y_0}) \quad (8)$$

by means of the empirical interpolation method (EIM) [3]. In a related work [6], Green's function for the Helmholtz operator was approximated by EIM, taking the wavenumber as a parameter. By plugging the approximation (8) into (6), the matrix $\mathbf{A}$ becomes affine in $\mathbf{p}$. The affine form carries over to the ROM (3), which can thus be evaluated at any values of $\mathbf{p}$ without performing operations of complexity N.

III. Numerical Example

This abstract presents results for two bodies and translational movement only. The general case will be given in the conference presentation. Fig. 1 shows a wireless power transfer system with variable vertical distance $t_v \in [8 \dots 12]$ mm.

The coils are wound from litz wire, which are treated by the homogenization method of [1]. The reference FE-BE model features 39,952 FE degrees of freedom (DoF) and 343 BE DoFs. The EIM is based on an adaptive greedy strategy on a dense sampling of the parameter domain with the error indicator

$$\frac{\|\tilde{\mathbf{G}} - \mathbf{G}\|_2}{\|\mathbf{G}\|_2} < 10^{-3}, \quad (9)$$

$$[\mathbf{G}] = G(\boldsymbol{x_0}, \boldsymbol{y_0}, \boldsymbol{p}) \qquad \text{for all pairs } (\boldsymbol{x_0}, \boldsymbol{y_0}). \quad (10)$$

In the present example, this procedure results in 7 coefficient matrices $\mathbf{A}_i$ in the approximation (5) to $\mathbf{A}$. To generate the ROM, a multi-point approach similar to [7] is used, employing the relative residual r as error indicator. For the termination criterion $r < 10^{-5}$, the ROM has the dimension $n = 16$.

Fig. 2 presents the coupling inductance L_{12} between the coils as a function of the distance t_v, at 101 equidistant sampling points. It was calculated by the FE-BE reference model, by the affine approximation to the BE part after (8), and by the ROM. The curves cannot be visually distinguished. Fig. 3 gives the relative error e,

$$e = \left| L_{12} - L_{12}^{\text{ref}} \right| / L_{12}^{\text{ref}}, \quad (11)$$

for the ROM with respect to the reference FE-BE system (total error) as well as with respect to the affine model. The total error lies below $2 \cdot 10^{-4}$, which is deemed sufficient for practical applications. The error introduced solely by the MOR

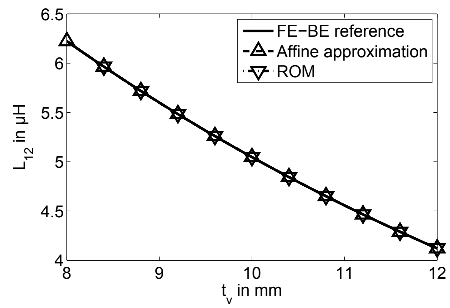
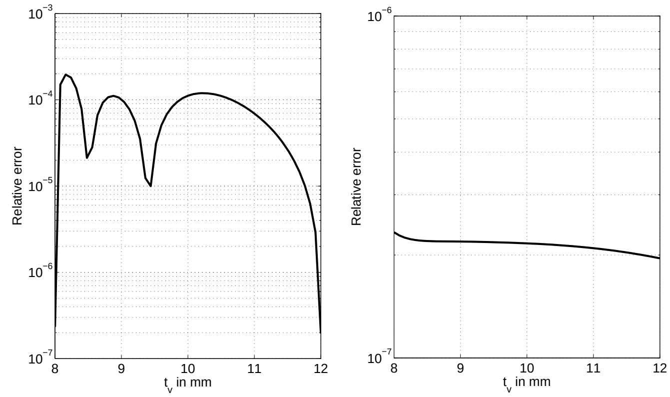


Fig. 2. Coupling inductance L_{12} as a function of vertical distance t_v.



(a) Error w.r.t. FE-BE reference. (b) Error w.r.t. affine approximation.

Fig. 3. Relative ROM error versus vertical distance t_v.

process lies in the range of 10^{-6}, indicating that the affine approximation is the dominant source of error.

The following runtime comparison demonstrates the computational efficiency of the ROM: Solving the FE-BE reference model takes 207 s excluding the time-consuming BE matrix assembly, whereas solving the ROM takes only 0.27 s, using prototype MATLAB code.

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